2.5 Average Rate of Change

Objectives

- Find the average rate of change of a function between two points.
- Find the slope of the secant line.

Average Rate of Change

If $y = f(x)$ then the ratio

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

is called the average rate of change of y with respect to x as x changes from x_1 to x_2.

Note: The average rate of change is the **slope between two points!**

Example 1:

For the function $y = 2x + 1$, find the average rate of change of y with respect to x, $\frac{\Delta y}{\Delta x}$,as x changes from 0 to 3.

Example 2:

What is the average rate of change of f(x) from x_1 =0.6 to x_2 = 8? Please write your answer rounded to the nearest hundredth.

$$f(x) = -7x^2 - 6x + 10$$

You try:

Find the average rate of change from $x_1 = 5.3$ to $x_2 = 9.1$ for the function $f(x) = 2x^2 - 6x + 9$.

$$\frac{\Delta y}{\Delta x} = \frac{f(9.1) - f(5.3)}{9.1 - 5.3} = \frac{120.02 \ - 33.38}{3.8} = 22.8$$

Example 3:

Let $f(x) = \mathrm{x}^2 - 1$. Find and simplify the expression that represents the average rate of change of f between x and $x + h$.

2.6 Instantaneous Rate of Change

Objectives

- Find the instantaneous rate of change of a function
- Interpret the meaning of f’

Slope of the secant line/tangent line

$$\frac{f(b) - f(a)}{b - a}$$

Instantaneous Rate of Change

The slope of a curve $f(x)$ at the point (a, b) is exactly **the slope of the tangent line** to the curve at that point. This is also called the instantaneous rate of change.

Notation:

For an input $x = a$ for a given function $f(x)$:

$f(a) =$ the height of the point, and

$f'(a) =$ the slope of the curve at the point.

I.E. f(x)= the y-coordinate, f ‘(x) = slope of the tangent line

Example: Interpret f(8)= 10 and f’(8) = -2.

Examples

1. $f(x)$ is the velocity in feet per hour a bulldozer is traveling at x hours. Which of the following does the slope $f'(x)$ represent?

2. Suppose $f(x)$ is the number of gallons of gas used by a car after it has traveled x miles. Suppose the car gets 23 miles/gallon. What is $f(115)$? Write the exact answer. Do not round.

Reminder-Difference Quotient

$$\frac{f(x+h)-f(x)}{h}$$

Example

- Calculate the difference quotients for $f\left(x\right) = 5 - 3x$ using $h = 0.1, 0.01,$ and 0.001. Use the results to approximate the slope of the tangent line to the graph of $f\left(x\right)$ at the point $\left(6, -13\right)$. If necessary, round the difference quotients to no less than six decimal places and round your final answer to two decimal places.

Graphically Example

- Find the slope of $f(x)$ at $x = -2$. The graph of $f(x)$ is shown below.
- Move the point on the curve to $x = -2$. Then plot two points on the tangent line. Finally, calculate the slope of $f(x)$ at $x = -2$. Write your answer as a simplified fraction or rounded to 4 decimal places.