Warm Up

Find the derivative:

1) Using the product rule $f(x) = (3x^2 - 1)(x^7 + 2)$

2) Using the quotient rule $h(x) = \frac{(x^2-3)}{x^3+1}$

3.2 Chain Rule and General Power Rule

The Chain Rule

If y = f(u) is a differentiable function of u and u = g(x) is a differentiable function of x, then y = f(g(x)) is a differentiable function of x and

$$\frac{dy}{dx} = \frac{dy}{du} * \frac{du}{dx}$$

Or equivalently

$$\frac{d}{dx}\left[f\left(g(x)\right)\right] = f'\left(g(x)\right)g'(x)$$

$$\frac{\mathrm{d}}{\mathrm{dx}}[\mathrm{f(g)}] = f'(g) * g'$$

Chain Rule

-Use when you have a function inside a function (composite function)

-Derivative of the outside (leave inside alone), then times the derivative of the inside

Example 1:

$$y = (2x - 1)^4$$

General power rule

If $y = [g(x)]^n$,then

$$\frac{dy}{dx} = n[g(x)]^{n-1} \cdot g'(x)$$

provided that $g'(x)$ exists.

Example 2

Find the derivative of $y = (x^2 + 1)^3$

Example 3: You Try!

Find the derivative $f(x) = (3x - 2x^2)^3$

Example 4:

Find the derivative $y = \sqrt[3]{(x^2 - 1)^2}$

Example 5:

Find the derivative $g(t) = -\frac{7}{(2t-3)^2}$

Example 6: Use Product and Chain Rule

Find the derivative

$$y = (3x^3 + 1)(-4x^2 - 3)^4$$

Example 7: Use Quotient and Chain Rule

Find the derivative

$$y = \frac{(x^3 + 4)^5}{3x^4 - 2}$$

Example 8

Find the equation of the tangent line at the point (2, 4) to the graph of

$$y = \sqrt[3]{(x^2 + 4)^2}$$

Hint: Start with the derivative, y'. Use that to find the slope at x =2. Then use the point (2, 4) to write the equation of a line.

Example 9:

From 2004 to 2013, the sales per share S (in dollars) for Apple can be modeled by $S = (0.0351t^2 - 0.134t + 1.19)^2 \quad for\ 4 \leq t \leq 13$
Where t is the year, with t = 4 corresponding to 2004

Use the model to approximate the rates of change in the sales per share in 2006, 2009, 2012.

Review of Basic Differentiation Rules

1. Constant Rule $\frac{d}{dx}[c] = 0$
2. Constant Multiple Rule $\frac{d}{dx}[cx] = c$
3. Power Rule $\frac{d}{dx}[x^n] = n(x)^{n-1}$
4. Product Rule $\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$
5. Quotient rule $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'g - fg'}{g^2}$
6. Chain Rule $\frac{d}{dx}[f(g(x))] = f'\big(g(x)\big) * g'(x)$