Warm up

Find the open intervals on which f is increasing or decreasing:

$$f(x) = x^2 - x - 4$$

$$f'(x) = 2x - 1$$

$$0 = 2x - 1$$

$$\frac{1}{2} = x = critical\ number$$

f(x) is increasing: (½ , ∞)

f(x) is decreasing: (-∞, ½)

Section 3.5 and 3.6

First Derivative Test, Extrema, and Absolute Extrema

Formal Definition of Relative Extrema

Let f be a function defined at c.

1. f(c) is a relative maximum of f when there exists an interval (a, b) containing c such that $f(x) \leq f(c)$ for all x in (a, b)

2. f(c) is a relative minimum of f when there exists an interval (a, b) containing c such that $f(x) \geq f(c)$ for all x in (a, b)

Relative Extrema - Graphically

We can think of relative extrema is as a “hill” or a “valley”, or as a peak. It does not have to be the lowest (highest) point overall, just the lowest (highest) relative to it’s surroundings!

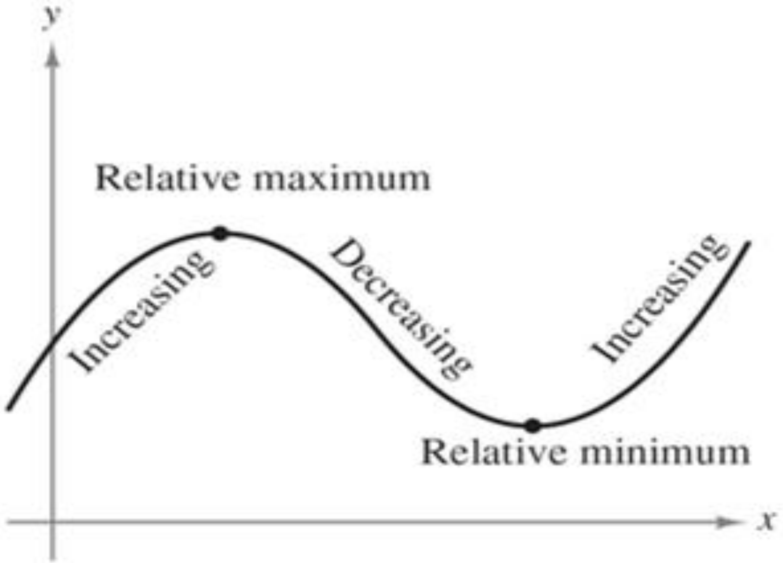


Figure 3.10

Critical Numbers

If f(c) is a relative extremum of f, then the relative extremum is said to occur at x = c, and x = c is a critical number of f.

If f(c) is a relative extrema, then $f'(c) = 0 \; or \; undefined.$

This means that critical numbers are possible locations of extrema!

We can use the signs of the first derivative to find where extrema occur. This is called the first derivative test.

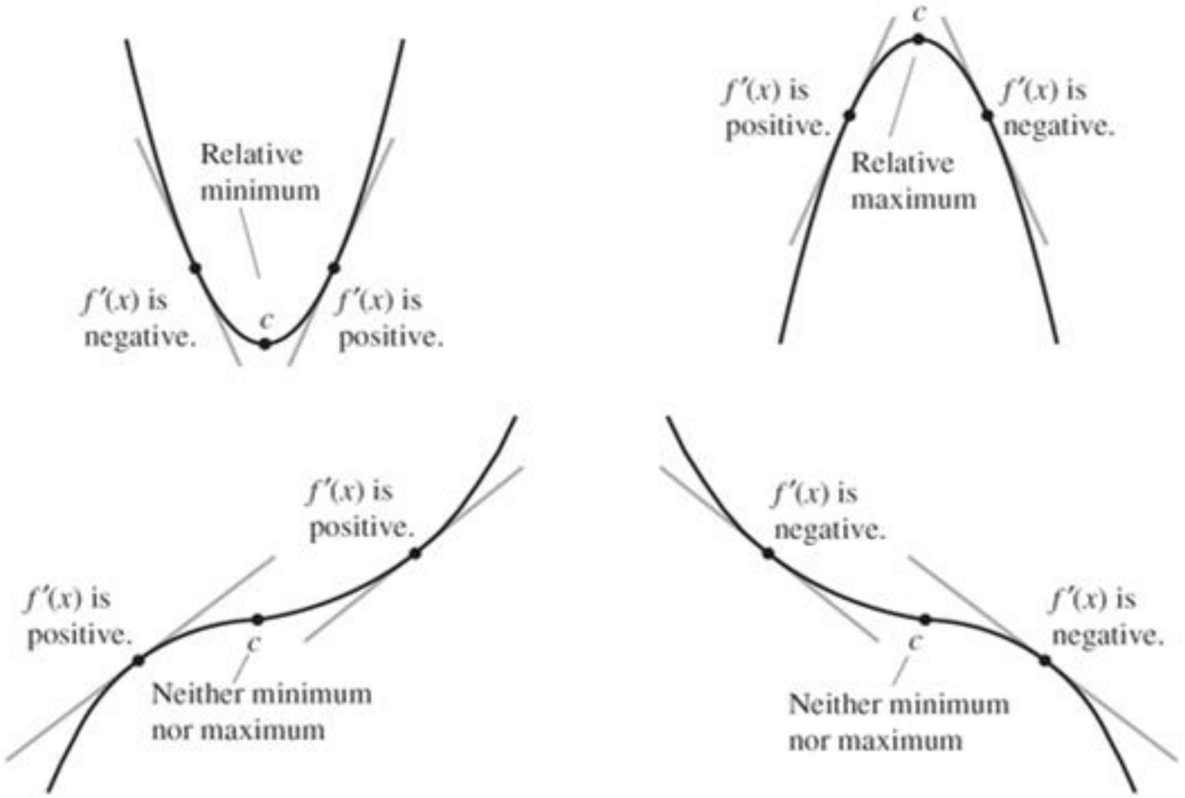
First Derivative Test:

- If f ’ goes from + to – , f(x) is going from increasing to decreasing; thus we have a maximum

- If f ’ goes from – to +, f(x) is going from decreasing to increasing; thus we have a minimum

- If f ’ does not change signs, then neither a min or max occurs.

Graphically:



Guidelines for finding Relative Extrema

1. Find the first derivative f ’
2. Set f ’= 0 or undefined and find the critical numbers.
3. Create a number line and test values in f ’ to find intervals that f(x) is increasing/decreasing.
4. If f goes from increasing to decreasing, we have a maximum. If f goes from decreasing to increasing, we have a minimum
5. Use the function f to find the y-value as needed.

Example 1:

Find all relative extrema for the function $f(x) = x^4 - x^3$

Example 2:

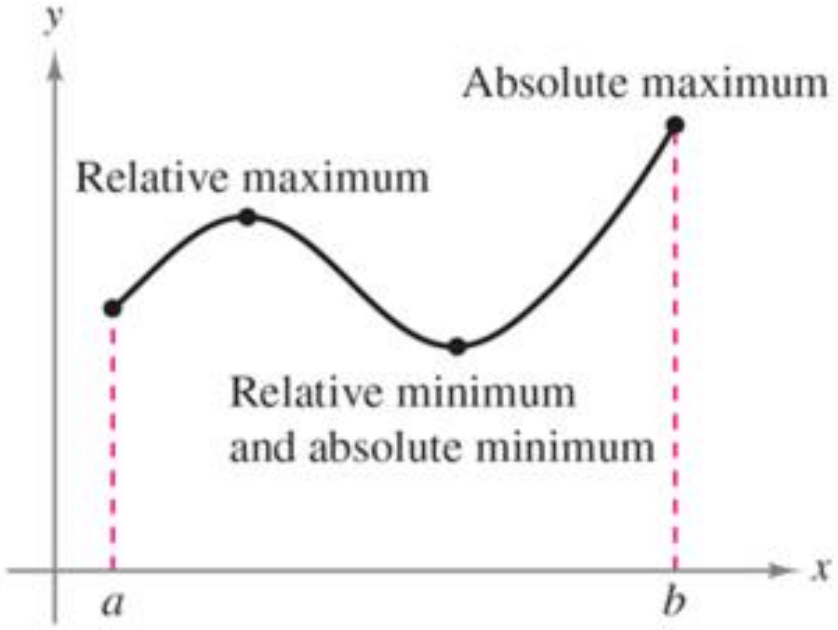
Find the relative extrema of $f(x) = \frac{x^4+1}{x^2}$

Example 3:

Find any relative extrema of the function $f(x) = \frac{x}{x+1}$

Absolute Extrema:

Absolute extrema is when the extrema is the highest or lowest point on the entire graph. Typically, we are going to look at absolute extrema only over a **closed interval,** as shown below!



Extreme Value Theorem

If f is continuous on the **closed** interval [a, b], then f has both an absolute minimum and an absolute maximum value on [a, b].

Guidelines for Finding Absolute Extrema

To find the absolute extrema of a continuous function f on a **closed** interval [a, b] use the following steps:

1. Find the critical numbers of f in the open interval (a, b)
2. Evaluate the function at each of its critical numbers in (a, b)
3. Evaluate f at each endpoint, a and b.
4. The least of these values is the minimum and the greatest is the maximum.

Example 4:

Find the absolute maximum and the minimum values on the interval [0, 5] of $f(x) = x^2 - 6x + 2$

Example 5:

Find the Extrema of f on the interval [-1, 2] of $f(x) = 3x^4 – 4x^3$

Example 6:

Find the extrema of f on the interval [-1, 3] of $f(x) = 2x - 3x^{\frac{2}{3}}$

Example 7:

We know that the fast-food restaurant's profit function from selling x hamburgers is given by $P = 2.44x - \frac{x^2}{20{,}000} - 5{,}000 \quad for\ 0 \le x \le 50{,}000$

Find the sales level that yields a maximum profit.