# Derivative Recap

Find the derivatives using the power rule:

$$f(x) = 5x - 2x^4 + x^{\frac{3}{2}} + \frac{7}{x}$$

# 3.1 Product and Quotient Rules

# The Product Rule

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

In a simpler way, we can also write this as

$$\frac{d}{dx}[fg] = f'g + fg'$$

# Example 1:

Find the derivative of $h(x) = (3x - 2x^2)(5 + 4x)$

# Example 2:

Find the derivative of $m(x) = \left(\frac{1}{x} + 1\right)(x - 1)$

# Example 3: You try!

Find the derivative of $y = (2x)(x^2 + 3x)$

Answer:

$$y' = (2)(x^2 + 3x) + (2x)(2x + 3)$$
$$y' = 2x^2 + 6x + 4x^2 + 6x = 6x^2 + 12x$$

# The Product Rule extended

The product rule can be extended to products that have more than two factors. For example, if f, g, and h are differentiable functions of x, then

$$\frac{d}{dx}[f(x)g(x)h(x)] = f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x)$$

For example 4: find the derivative of:

$$y = (x^2)(3x - 1)(4x + 5)$$

# The Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{\left(g(x)\right)^2}, provided\ g(x) \neq 0$$

In simpler form:

$$\frac{d}{dx}\left[\frac{f}{g}\right] = \frac{f'g - fg'}{g^2}$$

# Example 5:

Find the derivative of the function:

$$h(x) = \frac{3x + 4}{7x - 2}$$

# Example 6:

Find the equation of the tangent line at the point (1, -1) to the graph of

$$y = \frac{2x^2 - 4x + 3}{2 - 3x}$$

# Example 7: You try

Find the derivative using the quotient rule:

$$y = \frac{x^2 - 1}{2x + 3}$$

Answer:

$$y' = \frac{(2x)(2x+3) - (x^2-1)(2)}{(2x+3)^2} = \frac{4x^2 + 6x - 2x^2 - 2}{(2x+3)^2}$$

$$= \frac{2x^2 + 6x - 2}{(2x+3)^2}$$

# Homework Example

You are given that $f$ and $g$ are differentiable functions and that $f(5) = 10$ , $g(5) = -3$ ,$f'(5) = 1$ ,$g'(5) = 8$ .Determine $h'(5)$ given that:

a. $h(x) = \frac{f(x)}{g(x)}$

b. $h(x) = (f(x) + 3x - 1)\, g(x)$

# Revenue:

The demand function for a particular item is $D(x) = \frac{172}{5x+1}$. Find the marginal revenue when x = 5, rounding your answer to the nearest cent.

# Population

It is estimated that *t* years from now, a city will have a population modeled by $P(x) = (0.5t - 4)(0.4t + 6) + 60$. How fast will the population be growing in 19 years?