4.2 Second Derivative Test

Finding Extrema without the number line

Recall: Finding Relative Extrema

Recall our process for finding relative extrema:

1. Find f’
2. Set = 0 or undefined and solve for x, the critical numbers.
3. Create a number line and test values in f’ to determine signs
4. If f’ is +, f is increasing. If f’ -, f is decreasing.
5. Identify the max/min and Find y-coordinate if needed.

Using the Second derivative to find extrema

We will use first two steps:

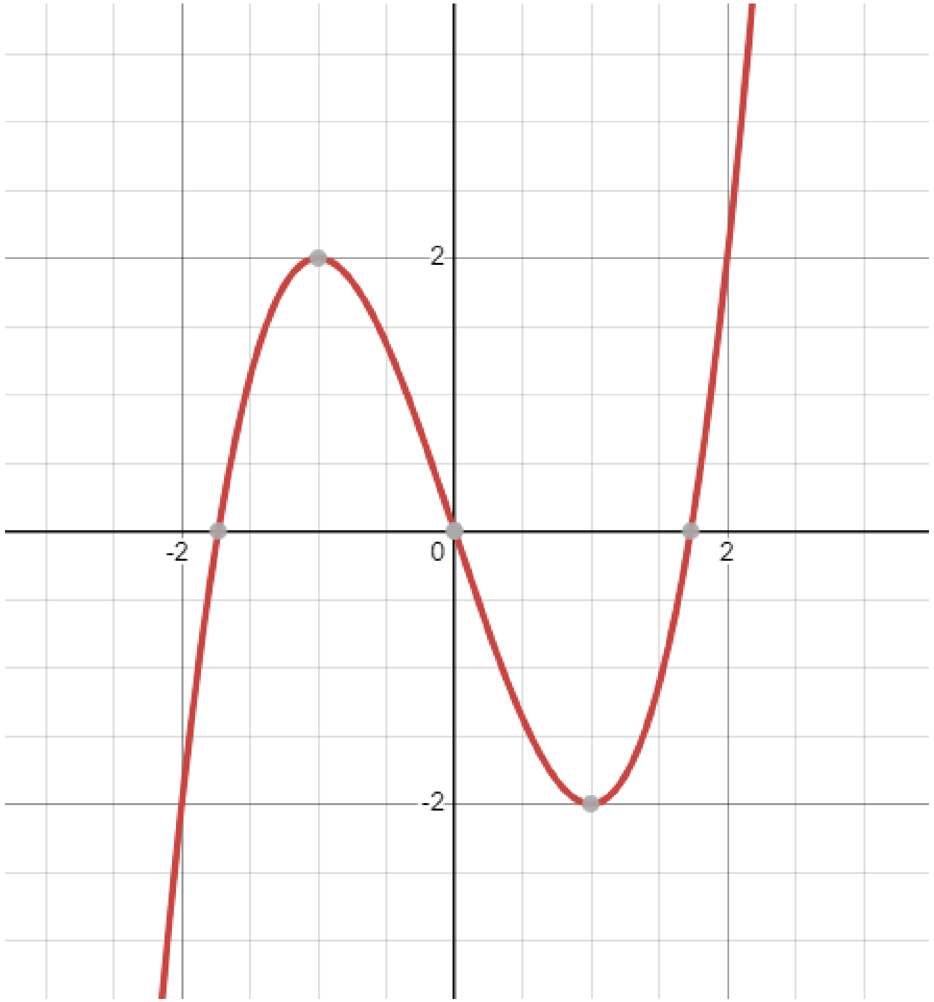
1. Find f’
2. Set = 0 or undefined and solve for x, the critical numbers

But now, we will use our second derivative to look at the concavity at those critical numbers!

3. Find f’’
4. Plug in critical numbers to f”.
   - If f” is +, f is concave up and thus critical number is a minimum.
   - If f” if -, f is concave down and thus critical number is a maximum.

Example 5:

Find extrema of $f(x) = x^3 - 3x$



Example 6:

Find all relative extrema of $f(x) = -3x^5 + 5x^3$ using the second derivative test.

Example 7: You try!

Find all relative extrema of the function using the second derivative test: $f(x) = x^4 + 8x^3 - 9$

Answer: Relative Max DNE, Relative min (-6, -44)

Example 8: Concept Question

Sketch a graph with the following characteristics:

$f(2) = f(4) = 0$

$f'(x) < 0$ if $x < 3$

$f'(3)$ is undefined

$f'(x) > 0$ if $x>3$

$f''(x) < 0$ for all $x \neq 3$

Example 9: Harder Derivative

Discuss the concavity: $f(x) = \frac{x^2}{x^2+1}$