Section 5.1

Exponential Functions

Exponential Function with Base b

The general form of an exponential function with base b is

$$f(x) = c \cdot b^{rx},$$

where b is a positive real number, $b \neq 1$, and c and r are real constants.

Ex: $f(x) = 2 \cdot 3^{0.5x}$

Properties of Exponential Functions

For any exponential function $f(x) = c \cdot b^{rx}$, the following properties are true.

1. The domain is the set of all real numbers.
2. The range is the set of all positive real numbers when $c > 0$.
3. The y-intercept is the point $(0, c)$.
4. The x-axis is a horizontal asymptote.
5. The function is continuous for all real x.
6. Suppose $b > 1$. Then, when $c = 1$ and $r = 1$, the slope is positive, and y is increasing and concave up.
7. Suppose $0 < b < 1$. Then, when $c = 1$ and $r = 1$, the slope is negative, and y is decreasing and concave up.

Example 1: Graphing an Exponential Function

Suppose $f(x) = 2^{3x}$ and $g(x) = \frac{1}{3}(2^{3x})$.

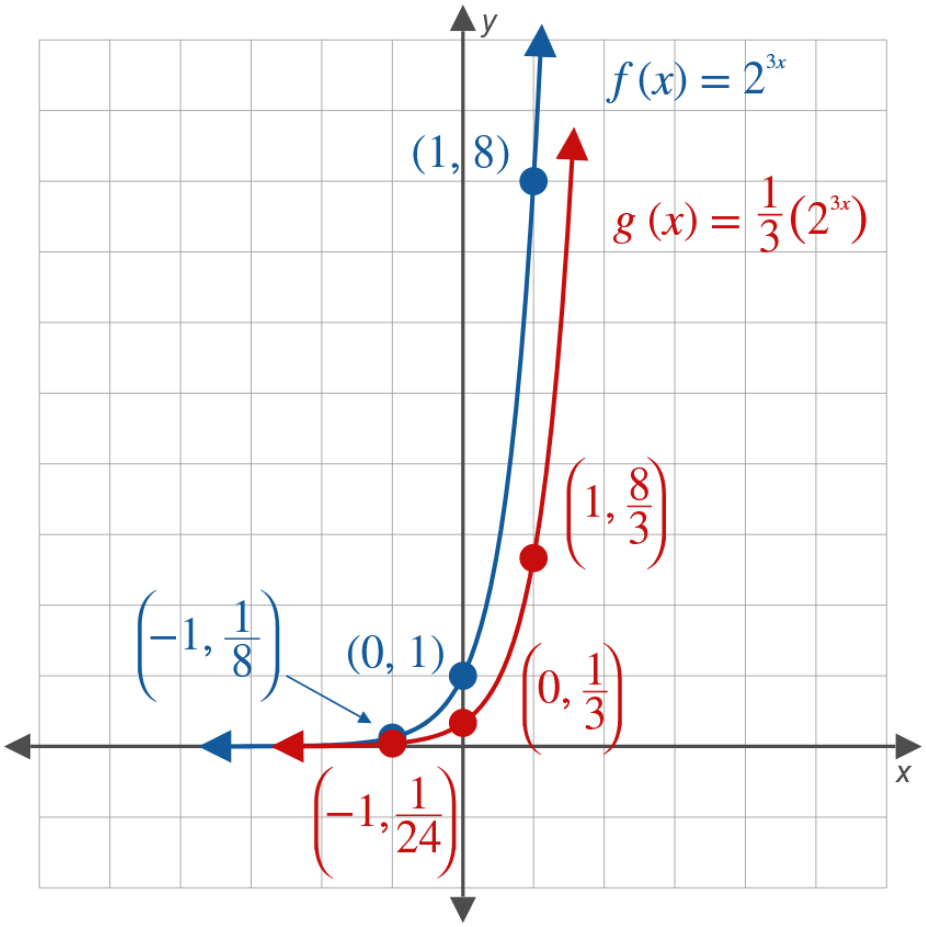
a. Find $f(-1)$, $f(0)$, and $f(1)$.

b. Find $g(-1)$, $g(0)$, and $g(1)$.

c. Sketch the graphs of both functions.

Example 1: Graphing an Exponential Function (cont.)

c.



Compound Interest Formula

For principal P invested at annual interest rate r (in decimal form) compounded n times per year for t years, the amount A is given by the following formula.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Example 2: Compound Interest

Suppose that $\$1000$ is invested at 12 percent compounded quarterly. What will be the value of the investment in

a. 1 year?

b. 3 years?

Natural Exponent:

In calculus, the most convenient choice for the base is the irrational number e, whose decimal approximation is $e \approx 2.71828182846$

The limit definition is as follows:

The irrational number e is defined to be

$$\lim_{x \to 0} (1 + x)^{\frac{1}{x}} = e$$

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Continuous Interest

For principal P invested at annual interest rate r (in decimal form) compounded continuously for t years, the amount A is given by

$$A = Pe^{rt}.$$

Example 3: Investment Value

If $\$5000$ is invested at 8 percent, what will be the value of the investment at the end of 5 years if the interest is compounded continuously?

Example 4: Initial Deposit Amount

Suppose that the grandparents of a child would like to make a one-time investment now to help pay for the child's college education in 20 years. How much should they deposit in an account earning 10 percent interest compounded continuously, so that the value of the account will accumulate to $\$20{,}000$?

Example

In a certain environment, a colony of bacteria grows according to the model $A = A_0 e^{0.12t}$, where A is the number of bacteria present at time t (in hours) after a culture is taken and A_0 is the initial number of bacteria present. If 250 bacteria are initially present, how many bacteria are present after 21 hours? Round to the nearest whole number.

Section 5.4

Derivatives of Exponential Functions

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Derivative of the Exponential Function

If $f(x) = e^x$, then

$$f'(x) = e^x.$$

Example 1: Finding the Derivative

a. Find $\frac{dy}{dx}$ for $y = x^3 + e^x$.

b. Find $\frac{dy}{dx}$ for $y = x^2 e^x$.

c. Find f’(x) for $f(x) = (x^2 - 2)(e^x + 3x)$

Chain Rule for Exponential Functions

If $f(x) = e^{g(x)}$, then

$$f'(x) = e^{g(x)} \cdot g'(x).$$

Example 2: Using the Chain Rule

Find $\frac{dy}{dx}$ if $y = e^{x^2+3x}$

Find $\frac{df}{dx}$ for $f(x) = e^{x^7-8x}$

Example: Find the equation of the tangent line.

$$f(x) = e^{3x-x^2} \text{ at the point } (3,1)$$

Example 3: Using the Chain Rule

For $f(x) = xe^{-x}$, find

a. $f'(x)$,

b. Find any critical values (where f ‘ = 0)

Example 4: Graphing Exponential Functions

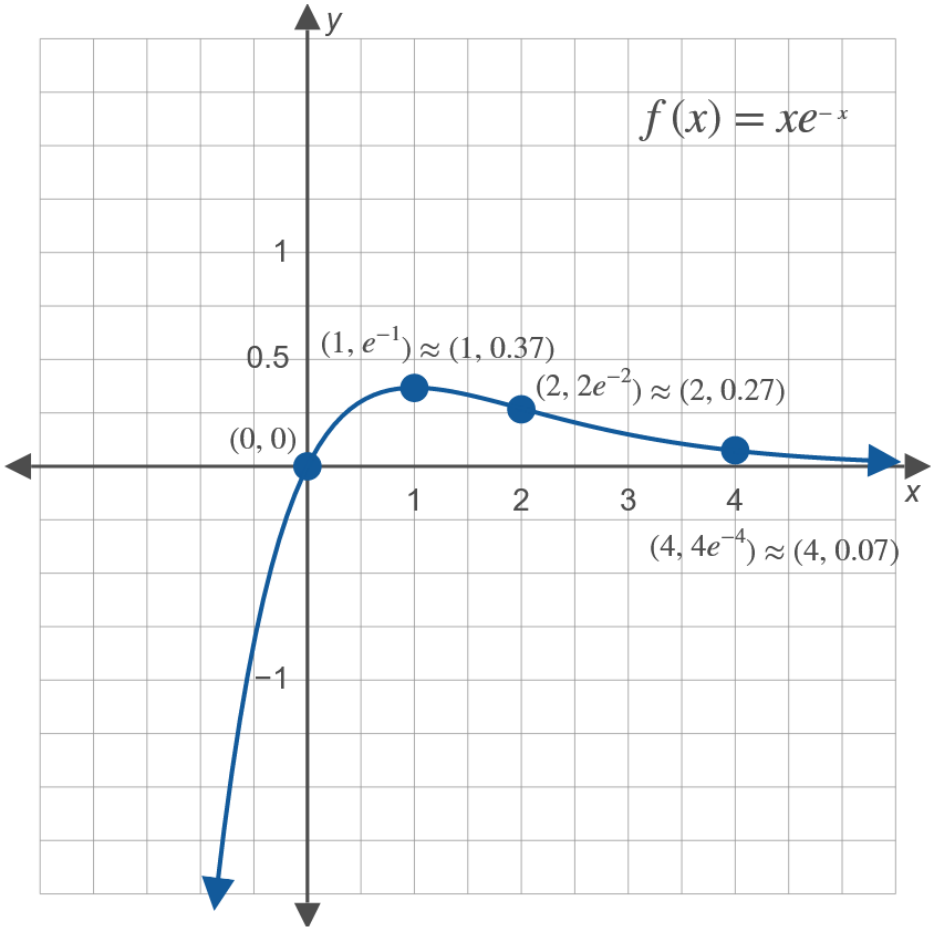
Let $f(x) = xe^{-x}$.

a) Find $f''(x)$

b) Find any hypercritical values (where f '' =0)

Example 4: Graphing Exponential Functions (cont.)

There is a point of inflection at $(2, 2e^{-2})$.



Example

Find the intervals on which the given function is increasing or decreasing.

$$y = e^{-0.8x^2-2}$$

Example

Find the slope of the tangent at x=2.

$$f(x) = \frac{-8e^x}{-5e^x - 2}$$

Example

Find the intervals on which the function is increasing or decreasing.

$$y = 6(4e^{5x} + 1)^3$$