Section 5.5

Growth and Decay

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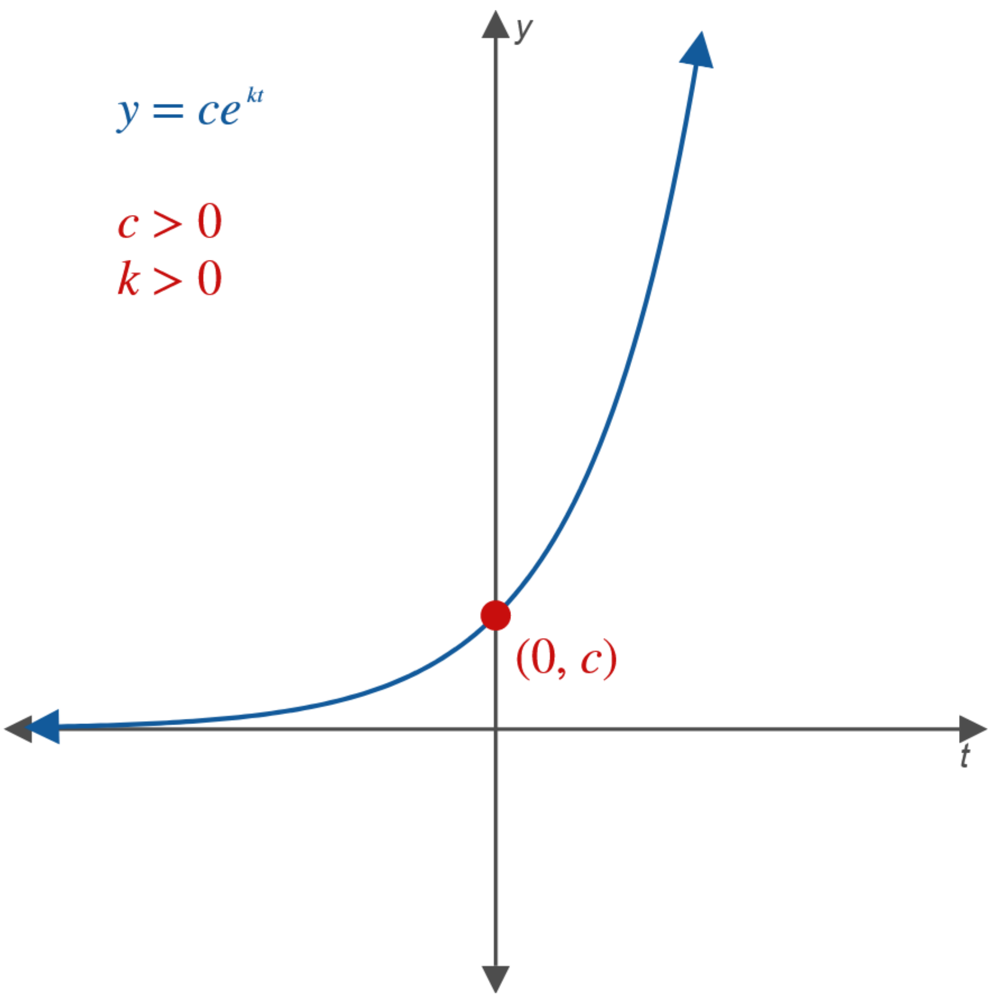
Exponential Growth



Exponential Growth:

$$y = ce^{kt}$$

Example

A population of squirrels grows exponentially at a rate of 4.1 percent per year. The population was 7700 in 1994.

a. Find the exponential function that represents the population t years after 1994.
b. What will the population by in the year 1999?
c. In what year will the population be 14,400? Round to the nearest year.

Example : Continuous Compounding Interest

Suppose that $2000 is invested in an account that earns 8 percent compounded continuously.

a. What will be the balance in 1 year?

b. When will the original investment be doubled?

Example

The demand for oil in a particular country doubles every 4 years. How long will it take for the demand to triple? Round to the nearest hundredth of a year.

Example

The decay rate for a radioactive isotope is 4.7 percent per year. Find the half life of the isotope. Round to the nearest tenth of a year.

Example : Population Growth

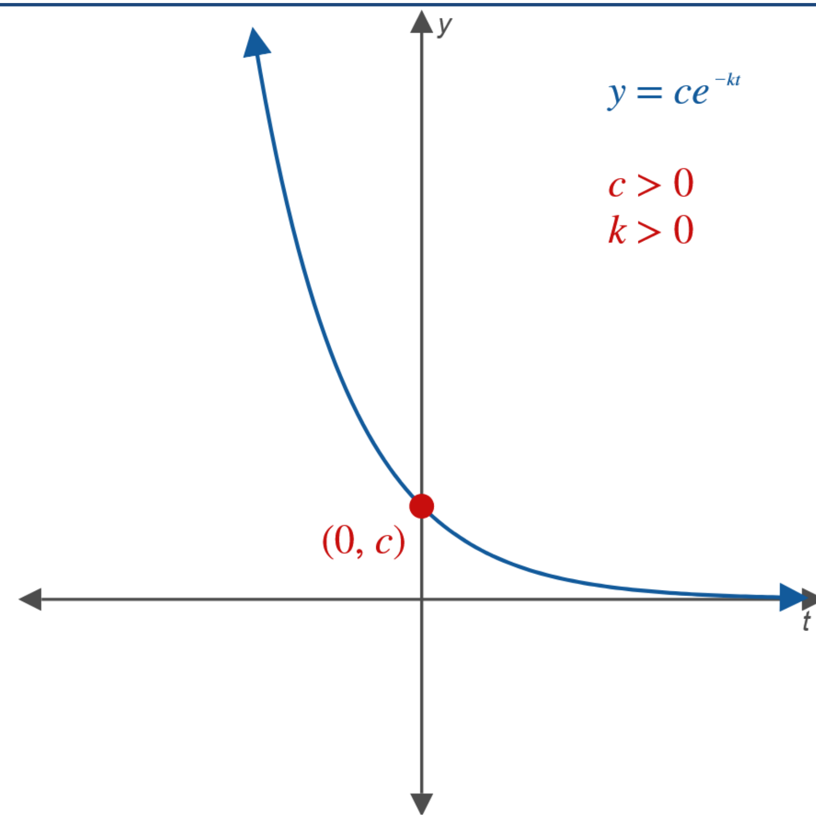
Suppose that a population of rabbits grows exponentially; that is, the rate of population growth depends on the size of the population. Further, suppose that on Monday at 8:00 a.m. there are 100 rabbits and that at 8:00 a.m. on Wednesday there are 130 rabbits.

Find an exponential function that represents the rabbit population at time t, where t is measured in days from Monday at 8:00 a.m.

Exponential Decline (Decay)

Exponential Decline:

$$y = ce^{-kt}$$



Example 3: Carbon-14

The radioactive substance carbon-14 is known to decay exponentially with respect to time after death. For carbon-14, the constant of proportionality is approximately $k = 0.000124$. Suppose that an animal bone is found that it contains 40 percent of its original amount of carbon-14. How old is the bone? Round to the nearest hundredth.

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Limited Growth

The mathematical model for **limited growth** is

$$y = c\left(1 - e^{-kt}\right),$$

where c and k are positive constants.

Section 5.6

Elasticity of Demand

Elasticity of Demand

If $p = D(x)$ is the demand function for a product, then the **elasticity of demand** for that product is

$$E = -\frac{1}{x} \cdot \frac{D(x)}{D'(x)}$$

Properties of Elasticity of Demand

1. If $E > 1$, then $R'(x) = D(x)\left(1 - \frac{1}{E}\right) > 0$, the revenue is increasing, and we say that the demand is **elastic**.

2. If $E < 1$, then $R'(x) = D(x)\left(1 - \frac{1}{E}\right) < 0$, the revenue is decreasing, and we say that the demand is **inelastic**.

3. If $E = 1$, then $R'(x) = 0$, the revenue is at its maximum, and we say that the demand has **unit elasticity**.

Example 1: Elasticity of Demand

Suppose that the demand function for a product is $p = D(x) = 300 - 2x$.

a. What is the price per unit if 50 units are sold?

b. Find the function describing the elasticity of demand E.

c. Find $E(50)$ and $E(110)$.

d. What quantity x maximizes revenue, R?

Example 2: Elasticity of Demand

A product is known to have a demand function $p = D(x) = 100e^{-0.5x}$.

a. Find the value of x for which $E = 1$.

b. Find the value for x that maximizes the revenue.

Note:

The answers for part a. and b. should be the same x-value.

Maximizing Revenue

Example 3:

A grocery store determines that the demand function for its bakery's bread is $x = 180 - 30p$, where x is the number of loaves of bread it sells daily and p is the unit price.

a. Find the quantity demanded when the price is $\$1.70$.

b. Find the function describing the elasticity as a function of p.

c. Find the elasticity at $p = \$1.70$.

d. Interpret the resulting elasticity.

e. Determine the revenue function R and find p so that R is a maximum.

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