Question 1

For each of the following statements, determine whether it is a proposition. Answer with "Yes" or "No."

1. The sum of two odd numbers is even. _____
2. y + 7 > 10 _____
3. The moon is made of cheese. _____
4. Have a great day! _____
5. This is a good book. _____

Question 2

Complete the truth table for the following logical expression and then determine if the expression is a tautology, contradiction, or neither:

$$\neg p \wedge (p \vee q)$$

p	q	$\neg p$	$p \vee q$	$\neg p \wedge (p \vee q)$

The expression is a: _____ **(tautology, contradiction, or neither)**

Question 3

Let $U = \{a, b, c, d, e, f\}$ be the universal set, with the following sets:

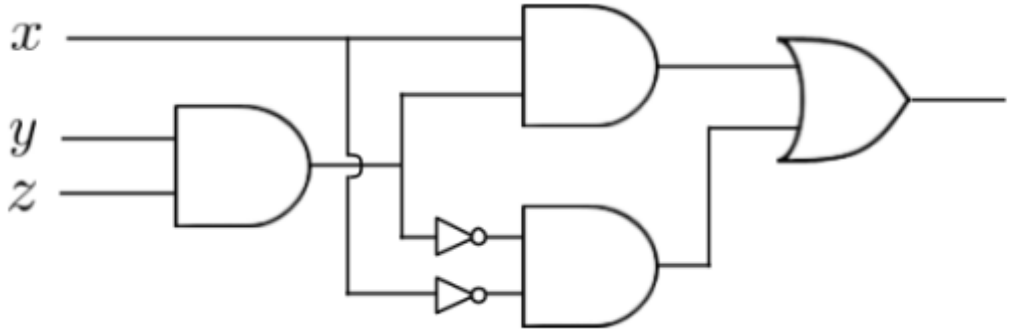
$$A = \{a, c, f\}, \quad B = \{b, c, d\}, \quad C = \{a, d, e\}$$

Find:

1. $B \cup C$
2. $A \cap C$
3. $B \cup (A \cap C)$
4. $C \cap \overline{B}$
5. $B - (A \cup C)$
6. $(A - B) \cap C$
7. $(\overline{A} \cap C) \cup B$
8. $P(A)$

Question 4

Fill in the input/output table from the following circuit:



x	y	z	f(x,y,z)
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

Question 5

Given that $x = 1$, $y = 1$, $z = 0$, and $w = 1$, evaluate each of the following Boolean expressions:

1. $\overline{yz} + w(x + \overline{z})$
2. $\overline{w} + x + yz$
3. $x\overline{y} + \overline{w}z + \overline{y}$

Question 6

$$B = \{5, \{3, 6\}, 6, \{2, 5\}, \{4\}, \{\emptyset\}\}$$

1.	$\lvert B\rvert =$	_____	
2.	$5 \in B$	*True*	*False*
3.	$\{3, 6\} \in B$	*True*	*False*
4.	$2 \in B$	*True*	*False*
5.	$\{4\} \in B$	*True*	*False*
6.	$3 \in B$	*True*	*False*
7.	$\emptyset \in B$	*True*	*False*
8.	$\{5\} \subseteq B$	*True*	*False*
9.	$\{3, 6\} \subseteq B$	*True*	*False*
10.	$\{6, 5\} \subseteq B$	*True*	*False*
11.	$\{4\} \subseteq B$	*True*	*False*
12.	$\emptyset \subseteq B$	*True*	*False*

Question 7

For the following input/output table, give a Boolean expression that is the sum of minterms and is equivalent to the function defined by the table.

x	**y**	**z**	$f(x, y, z)$
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

Question 8

Let $S(x, y)$ be the statement "student x has studied subject y", where the domain for x is the set of all students in a university, and the domain for y is the set of all subjects offered by the university. For each of the following English statements, **select** its equivalent logical expression.

English Statements

1. There is a student who has studied both Mathematics and Physics. _____
2. Every student has studied at least one subject. _____
3. There exists a subject that all students have studied. _____
4. No student has studied every subject. _____
5. Some student has studied every subject. _____
6. Every subject has been studied by at least one student. _____
7. No subject has been studied by all students. _____
8. Everyone who has studied Mathematics has also studied Physics. _____
9. Every student has studied both Mathematics and Physics. _____

Logical Expressions

A. $\forall x \exists y\, S(x, y)$

B. $\exists y \forall x\, S(x, y)$

C. $\neg(\exists y \forall x\, S(x, y))$

D. $\forall y \exists x\, S(x, y)$

E. $\forall x(S(x, \text{Mathematics}) \rightarrow S(x, \text{Physics}))$

F. $\exists x \forall y\, S(x, y)$

G. $\neg(\exists x \forall y\, S(x, y))$

H. $\forall x(S(x, \text{Mathematics}) \wedge S(x, \text{Physics}))$

I. $\exists x(S(x, \text{Mathematics}) \wedge S(x, \text{Physics}))$

Question 9

Prove that $\neg q \vee (\neg q \to p)$ is a tautology.

		Expression	**Law Used**
$\neg q \vee (\neg q \to p)$	$\equiv$		
	$\equiv$		
	$\equiv$		
	$\equiv$		
	$\equiv$		

Question 10

Prove the following argument using inference rules and the laws propositional logic:

$$\begin{array}{l} (p \vee r) \rightarrow q \\ q \rightarrow t \\ r \\ \hline \therefore t \end{array}$$

Step	Proposition	Justification
1		
2		
3		
4		
5		
6		